Solution **Ans. 1.5**

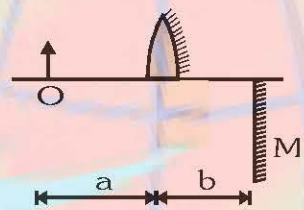
For case (a) $\frac{\mu}{v_1}-\frac{1}{\infty}=\frac{\mu-1}{R}$, $\frac{1}{v_2}-\frac{\mu}{v_1-R}=\frac{1-\mu}{\infty}$ and $v_2=\frac{2R}{m}$ $\Rightarrow$ m=$2(\mu-1)\mu$

For case (b) $\frac{1}{v_2}-\frac{\mu}{\infty}=\frac{1-\mu}{-R}$ and $v_2=\frac{R}{m-1}$ $\Rightarrow$ $\mu=m$

Therefore $\mu=2(\mu-1)\mu \Rightarrow \mu-1=\frac{1}{2} \Rightarrow \mu=1.5$

Example#27

In figure, L is half part of an equiconvex glass lens (μ = 1.5) whose surfaces have radius of curvature R = 40 cm and its right surface is silvered. Normal to its principal axis a plane mirror M is placed on right of the lens. Distance between lens L and mirror M is b. A small object O is placed on left of the lens such that there is no parallax between final images formed by the lens and mirror. If transverse length of final image formed by lens is twice that of image formed by the mirror, calculate distance 'a' in cm between lens and object.



Solution **Ans. 5**

Distance of image of object O from plane mirror = a + b. Since, there is no parallax between the images formed by the silvered lens L and plane mirror M, therefore, two images are formed at the same point. Distance of image = (a + 2b) behind lens. Since, length of image formed by L is twice the length of image formed by the mirror M and length of image formed by a plane mirror is always equal to length of the object, therefore, transverse magnification produced by the lens L is equal to 2.Since, distance of object from L is a, therefore, distance of image from L must be equal to 2a.

$$\therefore (a + 2b) = 2a \Rightarrow b = \frac{a}{2}$$

The silvered lens L may be assumed as a combination of an equi–convex lens and a concave mirror placed in contact with each other co–axially as shown in figure.

Focal length of convex lens f_1 is given by $\frac{1}{f_1}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \Rightarrow f_1$ = 40 cm

For concave mirror focal length, $f_m=\frac{R}{2}=-20$ cm

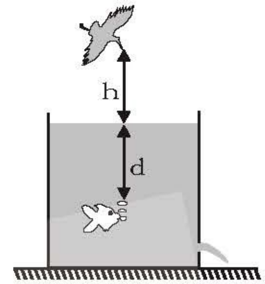
The combination L behaves like a mirror whose equivalent focal length F is given by

$$\frac{1}{F}=\frac{1}{f_m}-\frac{2}{f_1} \Rightarrow F=-10 \text{ cm},$$

Hence, for the combination u = –a, v = +2a, F = –10 cm

Using mirror formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{F} \Rightarrow a = 5$ cm

Distance of Bird as seen by fish $x_{Bf} = d + \mu h$



By differentiating $\frac{d(x_{fB})}{dt} = \frac{dh}{dt} + \frac{1}{\mu}\frac{d(d)}{dt}$

$v_{FB} \Rightarrow 3 + \frac{4}{\mu} = 6$ cm/s $\quad \left[\frac{dh}{dt} = 5 - 2 = 3\,\text{cm/s}, \frac{d(d)}{dt} = 2 - (-2) = 4\,\text{cm/s}, \mu = \frac{4}{3}\right]$

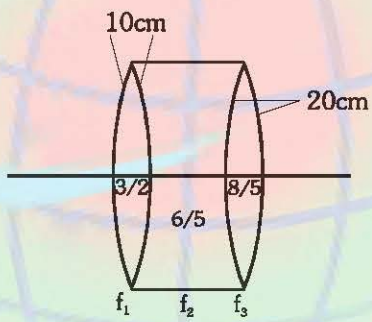
$v_{BF} = \left(\frac{d(d)}{dt}\right) + \mu\left(\frac{dh}{dt}\right) \Rightarrow 4 + \left(\frac{4}{3}\right)(3) \Rightarrow 8$ cm/s

$\frac{d(d)}{dt}$ (for fish image after reflection = 0) $\Rightarrow 3 + \frac{1}{\mu}(0) = 3$ cm/s

Similarly speed of image of bird $\Rightarrow$ 4 cm/s

Example#25

In the shown figure the focal length of equivalent system in the form of $\left(\frac{50x}{13}\right)$. Find the value of x.



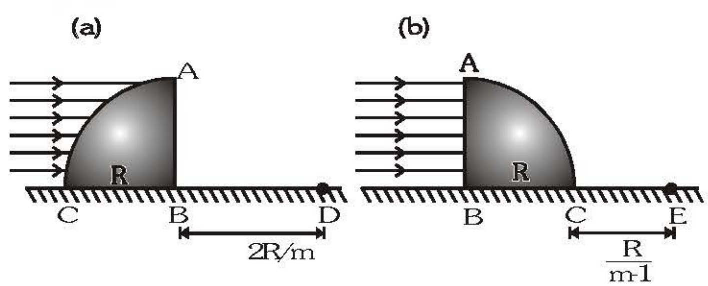
Ans. 2

$\frac{1}{f_1} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{10} + \frac{1}{10}\right) = \frac{1}{10}$; $\frac{1}{f_2} = \left(\frac{6}{5} - 1\right)\left(\frac{-1}{10} - \frac{1}{20}\right) = \frac{-3}{100}$ and $\frac{1}{f_3} = \left(\frac{8}{5} - 1\right)\left(\frac{1}{20} + \frac{1}{20}\right) = \frac{3}{50}$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} = \frac{1}{10} + \frac{-3}{100} + \frac{3}{50} = \frac{100}{13} = \frac{50x}{13} \Rightarrow x = 2$$

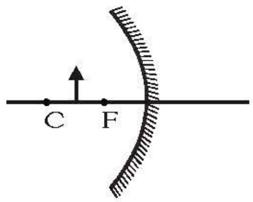
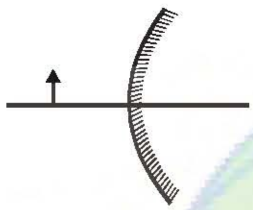
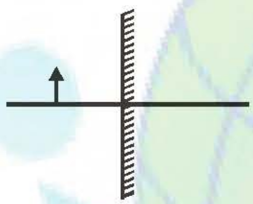
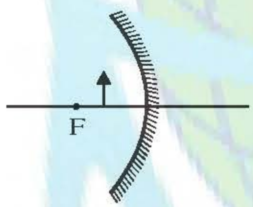
Example#26

Quarter part of a transparent cylinder ABC of radius R is kept on a horizontal floor and a horizontal beam of light falls on the cylinder in the two different arrangement of cylinder as shown in the figure (a) & (b). In arrangement (a) light converges at point D, which is at a distance 2R/m from B. And in arrangement (b) light converges at point E, which is at a distance R/(m – 1) from C. Find out the refractive index of the material.



Example#23

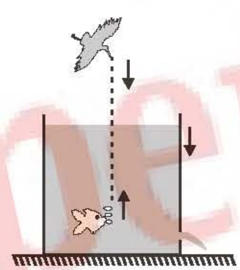
Column-I contains a list of mirrors and position of object. Match this with Column-II describing the nature of image.

Column I	Column II
(A) 	(P) real, inverted, enlarged
(B) 	(Q) virtual, erect, enlarged
(C) 	(R) virtual, erect, diminished
(D) 	(S) virtual, erect

Solution **Ans. (A) P (b)RS (C) S (D) QS**

Example#24

A bird in air is diving vertically over a tank with speed 5 cm/s, base of tank is silvered. A fish in the tank is rising upward along the same line with speed 2 cm/s. Water level is falling at rate of 2 cm/s. [Take : $\mu_{water} = 4/3$]



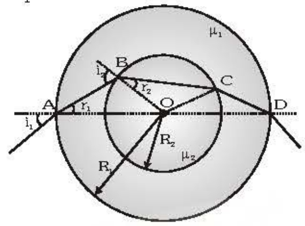
Column I (cm/s)		Column II	
(A)	Speed of the image of fish as seen by the bird directly	(P)	8
(B)	Speed of the image of fish formed after reflection in the mirror as seen by the bird	(Q)	6
(C)	Speed of image of bird relative to the fish looking upwards	(R)	3
(D)	Speed of image of bird relative to the fish looking downwards in the mirror	(S)	4

Solution **Ans. (A) – (Q) ; (B) – (R) ; (C) – (P) ; (D) – (S)**

Distance of fish asseen by bird $x_{fB} = h + \frac{d}{\mu}$

Example#19 to 21

There is a spherical glass ball of refractive index μ_1 and another glass ball of refractive index μ_2 inside it as shown in figure. The radius of the outer ball is R_1 and that of inner ball is R_2. A ray is incident on the outer surface of the ball at an angle i_1.



19. Find the value of r_1

(A) $\sin^{-1}\left(\frac{\sin i_1}{\mu_1}\right)$ (B) $\sin^{-1}(\mu_1 \sin i_1)$ (C) $\sin^{-1}\left(\frac{\mu_1}{\sin i_1}\right)$ (D) $\sin^{-1}\left(\frac{1}{\mu_1 \sin i_1}\right)$

20. Find the value of i_2

(A) $\sin^{-1}\left(\frac{R_2}{R_1}\frac{\sin i_1}{\mu_1}\right)$ (B)] $\sin^{-1}\left(\frac{R_1}{R_2}\frac{\sin i_1}{\mu_2}\right)$ (C) $\sin^{-1}\left(\frac{R_1}{R_2}\frac{\sin i_1}{\mu_1}\right)$ (D) $\sin^{-1}\left(\frac{R_2}{R_1}\frac{\sin i_1}{\mu_2}\right)$

21. Find the value of r_2

(A) $\sin^{-1}\left(\frac{R_1}{\mu_2 R_2}\sin i_1\right)$ (B) $\sin^{-1}\left(\frac{R_2}{\mu_2 R_1}\sin i_1\right)$ (C) $\sin^{-1}\left(\frac{R_1}{\mu_1 R_2}\frac{1}{\sin i_1}\right)$ (D) $\sin^{-1}\left(\frac{R_2}{\mu_1 R_1}\sin i_1\right)$

Solution

19. Ans. (A)

$\mu_1 \sin r_1 = \sin i_1 \Rightarrow r_1 = \sin^{-1}\left(\frac{\sin i_1}{\mu_1}\right)$

20. Ans. (C)

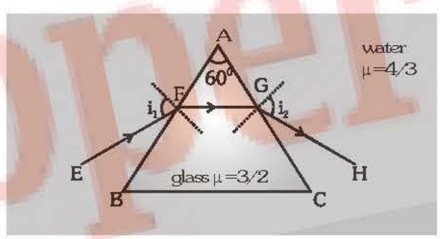
Using sine rule $\frac{\sin r_1}{R_2} = \frac{\sin(180 - i_2)}{R_1} \Rightarrow \sin i_2 = \frac{R_1}{R_2}\sin r_1 = \left(\frac{R_1}{R_2}\frac{\sin i_1}{\mu_1}\right) \Rightarrow i_2 = \sin^{-1}\left(\frac{R_1}{R_2}\frac{\sin i_1}{\mu_1}\right)$

21. Ans. (A)

$\frac{\mu_1}{\mu_2}\sin i_2 = \sin r_2$; $\frac{\mu_1}{\mu_2}\frac{R_1}{R_2}\frac{\sin i_1}{\mu_1} = \sin r_2$; $r_2 = \sin^{-1}\left(\frac{R_1}{\mu_2 R_2}\sin i_1\right)$

Example#22

Consider a an equilateral prism ABC of glass $\left(\mu = \frac{3}{2}\right)$ placed in water $\left(\mu = \frac{4}{3}\right)$



Column-I		Column-II	
(A)	FG is parallel to BC	(P)	Maximum deviation
(B)	$i_1 = 90^0$	(Q)	Minimum deviation
(C)	$i_1 = i_2 = \sin^{-1}\left(\frac{9}{16}\right)$	(R)	TIR will take place at surface AC
(D)	EF is perpendicular to AB	(S)	No TIR will take place at surface BC

Solution **Ans. (A) QS, (B) PS, (C) QS, (D) S**

At minimum deviation $i_1 = i_2$, EF$\|$ BC; At maximum deviation $i_1 = 90^0$ or $i_2 = 90^0$

For $i_1 = 0$, TIR will not take place at AC

Solution

13. Ans. (A)

$$\text{Time } t = \frac{\text{distance}}{\text{speed}} = \frac{\sqrt{Y_1^2 + x^2} + \sqrt{(\ell - x)^2 + Y_2^2}}{c}$$

14. Ans. (A)

For least time $\frac{dt}{dx} = 0 \Rightarrow \frac{2x}{\sqrt{Y_1^2 + x^2}} - \frac{2(\ell - x)}{\sqrt{(\ell - x)^2 + Y_2^2}} = 0 \Rightarrow \sin\theta_1 = \sin\theta_2 \Rightarrow \theta_1 = \theta_2$

15. Ans. (A)

Example#16 to 18

One hard and stormy night you find yourself lost in the forest when you come upon a small hut. Entering it you see a crooked old woman in the corner hunched over a crystal ball. You are about to make a hasty exit when you hear the howl of wolves outside. Taking another look at the gypsy you decide to take your chances with the wolves, but the door is jammed shut. Resigned to a bad situation you approach her slowly, wondering just what is the focal length of that nifty crystal ball.

16. If the crystal ball is 20 cm in diameter with R.I. = 1.5, the gypsy lady is 1.2 m from the ball, where is the image of the gypsy in focus as you walk towards her?

(A) 6.9 cm from the crystal ball
(B) 7.9 cm from the crystal ball
(C) 8.9 cm from the crystal ball
(D) None of these

17. The image of old lady is

(A) real, inverted and enlarged
(B) erect, virtual and small
(C) erect, virtual and magnified
(D) real, inverted and small

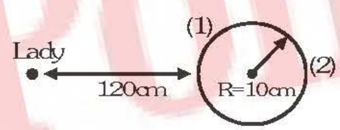
18. The old lady moves the crystal ball closer to her wrinkled old face. At some point you can no longer get an image of her. At what object distance will there be no change of the gypsy formed?

(A) 10cm (B) 5 cm (C) 15 cm (D) None of these

Solution

16. Ans. (A)

For refraction at 1st surface $\frac{1.5}{v_1} - \frac{1}{-120} = \frac{1.5 - 1}{+10} \Rightarrow v_1 = 36\text{cm}$



for refraction at 2nd surface $\frac{1}{v} - \frac{1.5}{(36 - 20)} = \frac{1 - 1.5}{-10} \Rightarrow v = \frac{80}{11.5} = 6.9\text{cm}$

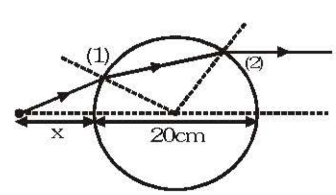
17. Ans. (D)

Total magnification $= m_1 m_2 = \left(\frac{\mu_1 v_1}{\mu_2 u}\right)\left(\frac{\mu_2 v}{\mu_1 v_1}\right) = \frac{v}{u}$ = negative

18. Ans. (B)

At this point image will formed at infinity

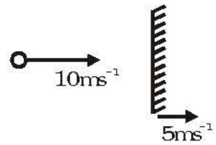
For refraction at second surface $\frac{1}{\infty} - \frac{1.5}{v_1} = \frac{1 - 1.5}{-10} \Rightarrow v_1 = -30\text{cm}$



For refraction at first surface $\frac{1.5}{-10} - \frac{1}{-x} = \frac{1.5 - 1}{+10} \Rightarrow x = 5\text{cm}$

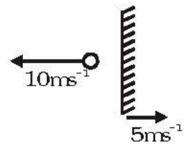
Solution **Ans.(AC)**

Case I



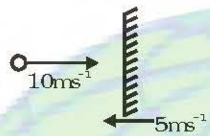
$\vec{v}_{om} = 5\hat{i}, \vec{v}_{Im} = -5\hat{i} \Rightarrow v_I = 0$

Case II



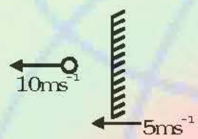
$\vec{v}_{om} = -15\hat{i}, \vec{v}_{Im} = +15\hat{i} \Rightarrow v_I = 20ms^{-1}$

Case II



$\vec{v}_{om} = 15\hat{i}, \vec{v}_{Im} = -15\hat{i} \Rightarrow v_I = 20ms^{-1}$

Case IV

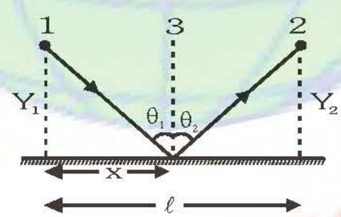


$\vec{v}_{om} = -5\hat{i}, \vec{v}_{Im} = +15\hat{i} \Rightarrow v_I = 0\ ms^{-1}$

OR $v_I = |2\vec{v}_m - \vec{v}_o| = |2(\pm 5) - (\pm 10)| = 20$ or 0 m/s

Example#13 to 15

A ray of light travelling with a speed c leaves point 1 shown in figure and is reflected to point 2. The ray strikes the reflecting surface at a distance x from point 1. According to Fermat's principle of least time, among all possible paths between two points, the one actually taken by a ray of light is that for which the time taken is the least (In fact there are some cases in which the time taken by a ray is maximum rather than a minimum).



13. Find the time for the ray to reach from point 1 to point 2.

(A) $\dfrac{\sqrt{Y_1^2 + x^2} + \sqrt{(\ell - x)^2 + Y_2^2}}{c}$ (B) $\dfrac{\ell}{c}$ (C) $\dfrac{Y_1}{c} + \dfrac{Y_2}{c}$ (D) $\dfrac{2x}{c}$

14. Under what condition is time taken least?

(A) $\theta_1 = \theta_2$ (B) $x = \ell - x$ (C) $Y_1 = Y_2$ (D) all of these

15. Which of the following statement is in accordance with Fermat's principle

(A) A ray as it moves from one point to another after reflection takes shortest possible path

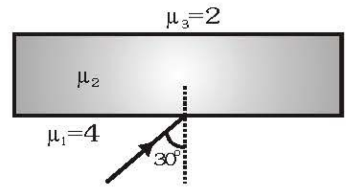
(B) A ray as it moves from one point to another after reflection takes longest possible path

(C) A ray as it moves from one point to another takes shortest possible time

(D) A ray as it moves from one point to another takes longest possible time

Example#10

A ray of light is incident in situation as shown in figure.

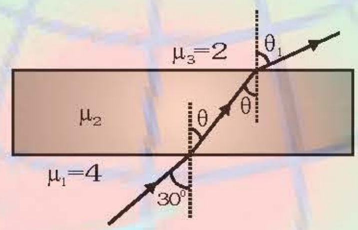


Which of the following statements is/are true?

(A) If $\mu_2 = 3.2$ then the angle of deviation is zero (B) If $\mu_2 = 2.8$ then the angle of deviation is 60^0

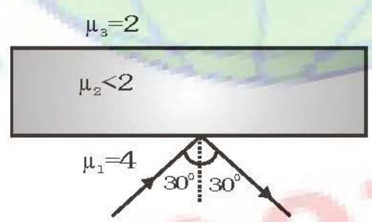
(C) If $\mu_2 = 1.8$ then the angle of deviation is 120^0 (D) If $\mu_2 = 1.8$ then the angle of deviation is 60^0

Solution **Ans.(BC)**



$\mu_1 \sin 30^0 = \mu_2 \sin\theta = \mu_3 \sin\theta_1 \Rightarrow 2 = \mu_2 \sin\theta = 2 \sin\theta_1 \Rightarrow \theta_1 = 90^0$ and $\mu_2 > 2$

For $\mu_2 < 2$, TIR will take place at first surface.



$\Rightarrow \delta = 120^0$

Example#11

A fish lies at the bottom of a 4m deep water lake. A bird flies 6 m above the water surface and refractive index of water is 4/3. Then the distance between

(A) Bird and image of fish is 9 m (B) Fish and image of bird is 12 m

(C) Fish and image of bird is 8m (D) Fish and image of bird is 10m

Solution **Ans. (AB)**

For a bird, fish appears 3 m below the water surface and for fish, bird appears 9m above the surface.

Example#12

A plane mirror and an object has speeds of 5 m/s and 10 m/s respectively. If the motion of mirror and object is along the normal of the mirror then the speed of image may be :–

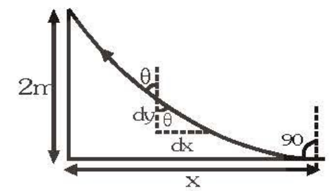
(A) 0 m/s (B) 10 m/s (C) 20 m/s (D) 25 m/s

Example#7

A man of height 2 m stands on a straight road on a hot day. The vertical temperature in the air results in a variation of refractive index with height y as $\mu = \mu_0\sqrt{(1+ay)}$ where μ_0 is the refractive index of air near the road and a=2 × 10^{-6}/m. What is the actual length of the road, man is able to see

(A) 2000 m (B) 390 m (C) infinite distance (D) None of these

Solution **Ans. (A)**

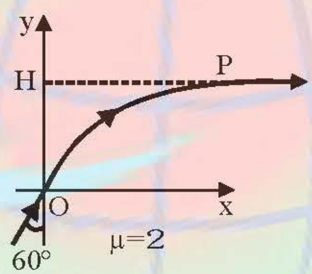


$\mu \sin\theta = \mu_0 \sin 90 = \mu_0 \Rightarrow \sin\theta = \frac{\mu_0}{\mu} = \frac{1}{\sqrt{1+ay}}$

But $\frac{dx}{dy} = \tan\theta \Rightarrow dx = (dy)\left(\frac{1}{\sqrt{ay}}\right) \Rightarrow x = \frac{1}{\sqrt{a}}\int_0^2 \frac{1}{\sqrt{y}}dy = \left[2\sqrt{\frac{y}{a}}\right]_0^2 = 2000\text{m}$

Example#8

A system of coordinates is drawn in a medium whose refractive index varies as $\mu = \frac{2}{1+y^2}$, where $0 \le y \le 1$ and $\mu = 2$ for $y < 0$ as shown in figure. A ray of light is incident at origin at an angle 60 with y–axis as shown in the figure. At point P ray becomes parallel to x-axis. The value of H is :-



(A) $\left\{\left(\frac{2}{\sqrt{3}}\right)-1\right\}^{1/2}$ (B) $\left\{\frac{2}{\sqrt{3}}\right\}^{1/2}$ (C) $\left\{(\sqrt{3})-1\right\}^{1/2}$ (D) $\left(\frac{4}{\sqrt{3}}-1\right)^{1/2}$

Ans. (A)

$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2 \Rightarrow$ at origin x =0, y = 0 $\Rightarrow \mu = 2$ & $\theta = 60$

at point P : $\theta = 90 \Rightarrow 2\sin 60 = \mu \sin 90 \Rightarrow \sqrt{3} = \frac{2}{y^2+1} \Rightarrow y = \left[\left(\frac{2}{\sqrt{3}}\right)-1\right]^{1/2}$

Example#9

A ray of light is incident along a vector $\tilde{i}+\tilde{j}-\tilde{k}$ on a plane mirror lying in y–z plane. The unit vector along the reflected ray can be

(A) $\frac{\tilde{i}+\tilde{j}-\tilde{k}}{\sqrt{3}}$ (B) $\frac{\tilde{i}-\tilde{j}+\tilde{k}}{\sqrt{3}}$ (C) $\frac{-\tilde{i}+\tilde{j}-\tilde{k}}{\sqrt{3}}$ (D) $\frac{3\tilde{i}+\tilde{j}-\tilde{k}}{\sqrt{3}}$

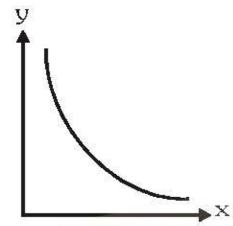
Solution **Ans. (C,D)**

According to law of reflection $\tilde{r} = \tilde{e} - 2(\tilde{e}.\tilde{n})\tilde{n}$ Here $\tilde{e} = \frac{\tilde{i}+\tilde{j}-\tilde{k}}{\sqrt{3}}$, $\tilde{n} = \pm\tilde{i}$

so $\tilde{e}.\tilde{n} = \frac{1}{\sqrt{3}} \Rightarrow \tilde{r} = \frac{\tilde{i}+\tilde{j}-\tilde{k}}{\sqrt{3}} \pm \frac{2\tilde{i}}{\sqrt{3}} = \frac{3\tilde{i}+\tilde{j}-\tilde{k}}{\sqrt{3}}$ or $\frac{-\tilde{i}+\tilde{j}-\tilde{k}}{\sqrt{3}}$

Example#5

If x and y denote the distances of the object and image from the focus of a concave mirror. The line y= 4x cuts the graph at a point whose abscissa is 20 cm. The focal length of the mirror is



(A) 20 cm (B) 40 cm (C) 30 cm (D) can't be determined

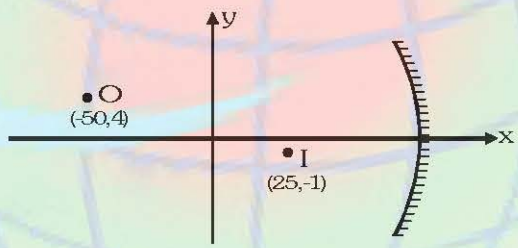
Solution **Ans.(B)**

For x = 20 cm, y = 4 20 = 80 cm

From Newton's formula $xy = f^2 \Rightarrow (2)(80) = f^2 \Rightarrow f = 40$ cm

Example#6

A concave mirror forms an image I corresponding to a point object O. The equation of the circle intercepted by the xy plane on the mirror is



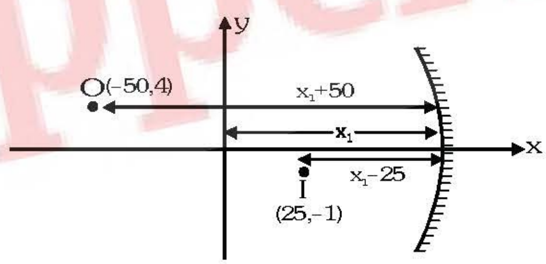
(A) $x^2 + y^2 = 1600$ (B) $x^2 + y^2 - 20x - 1600 = 0$

(C) $x^2 + y^2 - 20x - 1500 = 0$ (D) $x^2 + y^2 - 20x + 1500 = 0$

Solution **Ans. (C)**

From mirror equation $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}=\frac{1}{-(x_1-25)}+\frac{1}{-(x_1+50)}=\frac{1}{f}$ and

from $m=-\frac{v}{u}=-\frac{1}{4}$; $\frac{x_1-25}{x_1+50}=\frac{1}{4} \Rightarrow 4x_1-100 = x_1+50 \Rightarrow 3x_1 = 150 \Rightarrow x_1 = 50$ unit



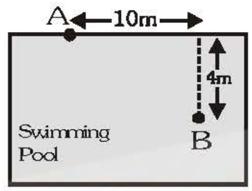
$$\frac{1}{f}=\frac{-1}{50-25}+\frac{-1}{50+50}=-\left(\frac{1}{25}+\frac{1}{100}\right) \Rightarrow f = -20 \text{ unit} \Rightarrow R = -40 \text{ unit}$$

Centre of circle will be at (10,0)

Equation of required circle $(x-10)^2 + (y-0)^2 = (40)^2 \Rightarrow x^2 + y^2 - 20x - 1500 = 0$

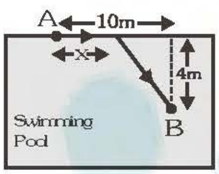
Example#3

On one boundary of a swimming pool, there is a person at point A whose speed of running on ground (boundary) is 10 ms^{-1} , while that of swimming is 6 ms^{-1}. He has to reach a point B in the swimming pool.The distance covered on the boundary so that the time required to reach the point B in the pool is minimum, is–



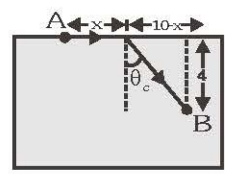
(A) 10m (B) 6m (C) 7m (D) $\sqrt{116}$ m

Solution **Ans. (C)**



Required time $t=\frac{x}{10}+\frac{\sqrt{4^2+(10-x)^2}}{5}$ For minimum time $\frac{dt}{dx}=0 \Rightarrow x= 7m$

OR



Just like light, he has different speeds on ground and in water, so to minimize the time, Fermat's principle must hold good.

$$\sin\theta_C=\frac{6}{10}=\frac{3}{5}\Rightarrow\theta_C=37^0\Rightarrow\tan\theta_C=\frac{3}{4}=\frac{10-x}{4}\Rightarrow x=7m$$

Example#4

A person has D cm wide face and his two eyes are separated by d cm. The minimum width of a mirror required for the person to view his complete face is

(A) $\frac{D+d}{2}$ (B) $\frac{D-d}{4}$ (C) $\frac{D+d}{4}$ (D) $\frac{D-d}{2}$

Solution **Ans. (D)**

According to ray diagram :

$$H_1M'_1=\frac{H_1E_2}{2} \;\&\; H_2M'_2=\frac{H_2E_1}{2}$$

$$H_1E_2=D-\frac{1}{2}(D-d)=\frac{D+d}{2}=H_2E_1$$

$$M'_1M'_2=D-H_1M'_1-H_2M'_2=D-\left(\frac{D+d}{2}\right)$$

$$=\frac{D-d}{2}$$